

Development of a Self-Check System for Mental Health using a Pulse Wave Mouse

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Abstract. We applied nonlinear chaos analysis to fingertip pulse wave data and found that the Lyapunov exponent, a measure of the “divergence” of the trajectory of the attractor, was a useful index of mental health in humans, particularly for the early detection of dementia and depressive psychosis, and for monitoring mental changes in healthy persons. We developed an easy-to-use economical device, a PC mouse with an integrated sensor for measuring the pulse waves, and its required software, to make the measurements. After about 1 min of measurement, the Lyapunov exponent is calculated and displayed as a graph on the PC. An advantage of this system is that the measurements can be made very easily and mental health can be assessed when the PC is not being used for other work. Moreover, the measured data can be saved according to the time and date, so diurnal changes and changes over longer time periods can be monitored as a time series. The changes in the Lyapunov exponent are displayed on the PC as constellation graphs.

1 Introduction

In Japan, the annual number of suicides has reached 30,000 in each of the past 3 years, with many being caused by serious depressive psychosis. Japan has world’s highest life expectancy, and dementia has increased along with the aging population. This has become a social problem, and appropriate handling by the family of the patient and society is essential for both depressive psychosis and dementia. Quite often, however, these ailments advance without the patient being aware of the condition, and thus early detection and treatment are necessary.

Routine daily checking of the person's behaviour and condition is necessary for detecting depressive psychosis and dementia. Subjective observations alone are not sufficient, and objective data must be gathered using scientific methods. Previously used methods include electroencephalography and diagnostic brain imaging, both of which require a high level of technical ability and knowledge. These are not convenient measurement methods when the time and cost are considered, and thus simple and economical methods are needed. We earlier demonstrated that the "divergence" of the attractor trajectory obtained by nonlinear analysis of fingertip pulse waves was a useful index for assessing mental health. It was particularly effective in the detection of dementia and the diagnosis of depressive psychosis [1]. With this technique, the pulse wave data, which have chaotic behaviour, are measured for a few minutes. The data are subjected to nonlinear analysis and the largest Lyapunov exponent, which is a measure of the divergence of the attractor trajectory, is calculated. In addition, patients are asked to answer some simple questions and record simple statements about their condition at the time of taking the measurements to encourage self-reporting of their physical health status. The personal ID code, date and time of measurement, fingertip pulse wave data, and the patient responses and a brief statement about their condition at the time of measurement are stored as time series data to create a database. The user searches the database for data pertaining to multiple measurement time points and displays the change in divergence with time or date on a constellation graph that we have devised. The changes in mental health and the self-reported status at the time of measurement are fed back to the subject, whose pulse waves were measured, making self-assessment possible. In Section 2, we explain the method of calculating the "divergence value" by applying the nonlinear analysis of fingertip pulse wave data and the self-check details. In Section 3, we discuss the relevance of using fingertip pulse wave data and the relationship between the "divergence value" and the psychology of dementia. The mental health self-check system is outlined in Section 4 and the effectiveness and shortcomings of the system, along with future plans, are discussed in Section 5.

2 . Calculation of the "divergence" value of fingertip pulse waves and details of self-checking

2.1 Method of measuring fingertip pulse waves

Fingertip pulse waves, which are linked to the bioinformation network, were measured to determine the divergence. Fingertip pulse waves have been found to have chaotic characteristics [2], [3], [5]. Changes in the amount of haemoglobin flowing through the capillaries can be measured by irradiating them with an infrared beam, as shown in Figure 1. The waveform is naturally synchronised with the beating of the heart, but the information is different from the cardiac wave. The changes in haemo-

globin level caused by the constriction of capillaries in the fingertip constitute time series data of a complex system that include information from the sympathetic and parasympathetic nerves. Pulse wave data are collected at a frequency of 200 Hz and a resolution of 12 bits. The measurement duration is variable, depending on the experimental conditions. Time series data consisting of 12,000 points could be obtained in 1 min of measurement.



Fig. 1. Measurement of pulse waves through infrared irradiation of capillaries. Frequency 200 Hz, resolution 12 bits.

The change in haemoglobin level in the capillaries is believed to be related to the baroreceptors, which are linked to the sympathetic and parasympathetic nerves via the brain stem and the spinal cord, as shown in Figure 2. This is a convenient method because it does not require special care with regard to the room temperature, place of measurement, and measurement conditions. Moreover, because the measurement time is very short, the data collection is not a burden to the subject.

2.2 Chaos analysis of the pulse wave and calculation of divergence (Lyapunov exponent) of the attractor trajectory

Figure 3 is a flow diagram showing the procedure, from the measurement of pulse waves to calculation of the Lyapunov exponent.

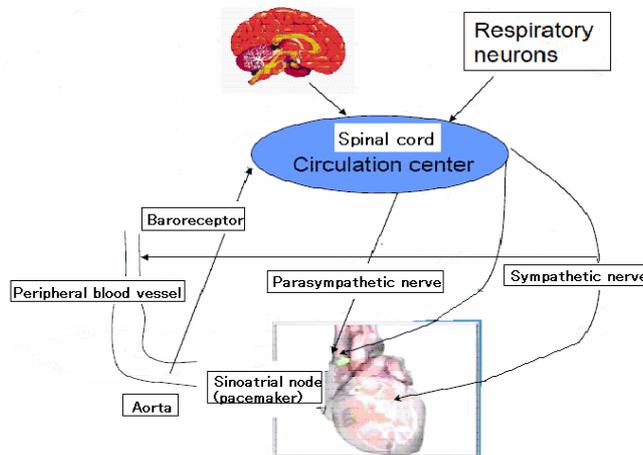


Fig. 2. A conceptual diagram showing the flow of information from the brain stem to peripheral blood vessels through the action of sympathetic and parasympathetic nerves.

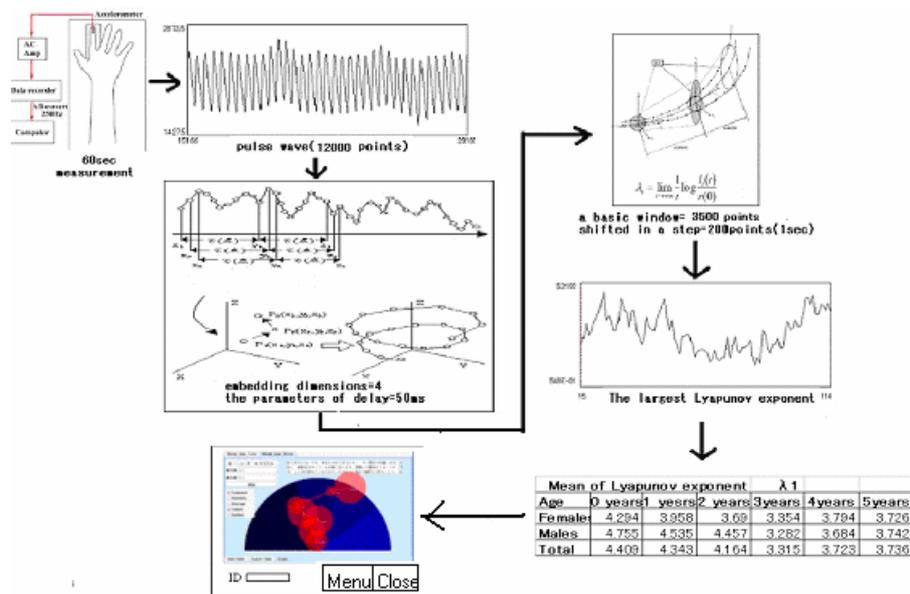


Fig. 3 Flow diagram showing the procedure from the measurement of pulse waves to calculation of the Lyapunov exponent.

To construct the attractor, we set a delay time and the number of embedding dimensions according to Tarkens [10][11]. We used four embedding dimensions and a delay time of 50 msec [4]. Figure 3 illustrates the method of embedding in three-dimensional phase space. Although effective information can be obtained from the shape of the four-dimensional attractor, we calculated the Lyapunov exponent, which is an index of trajectory instability and a characteristic of chaos.

Our method of calculating the Lyapunov exponent is explained in Figure 4, using three-dimensional phase space as an example. We assumed that a small sphere (hypersphere) of radius ε is the initial value for a three-dimensional chaotic dynamic system. After being mapped once, the sphere was stretched in the e_1 direction and compressed in the e_3 direction, and assumed the shape of an ellipsoid, as shown in Figure 4. We designated the logarithm of the expansion rate per unit time along the directions e_1 , e_2 , and e_3 as λ_1 , λ_2 , and λ_3 , respectively. Here, λ_1 , λ_2 , and λ_3 are the Lyapunov exponents and their set is the Lyapunov spectrum. We repeated the procedure of representing this hypersphere as many times as the calculation was iterated. After that, their sum was taken and the average was determined to calculate the Lyapunov spectrum. Because four embedded dimensions were set as the optimum number of dimensions for the pulse wave, we obtained the four Lyapunov exponents, λ_1 , λ_2 , λ_3 , and λ_4 , as the Lyapunov spectrum. Amongst these, the largest Lyapunov exponent, λ_1 , was used in the calculation to prepare the constellation graphs.

The calculation equations for the Lyapunov exponent are shown below. For the time series data $x(i)$, with $I = 1, \dots, N$ obtained from the fingertip pulse waves, the phase space was reconstructed using the method of time delays. Assuming that we

create a d-dimensional phase space using a constant time delay τ , the vectors in the space are generated as d-tuples from the time series and are given by

$$\mathbf{X}(i) = (x(i), \dots, x(i - (d - 1)\tau)) = \{x_k(i)\} \quad (1)$$

where $x_k(i) = x(i - (k - 1)\tau)$, with $k = 1, \dots, d$. To reconstruct the phase space correctly, the parameters of delay (τ) and embedding dimensions (d) should be chosen optimally [4]. In time series data recorded from human finger photoplethysmograms, we chose the parameters $\tau = 50$ msec and $d = 4$, as in references [2] and [3].

In the reconstructed phase space, one of the important measures of complexity is the largest Lyapunov exponent λ_1 . If $\mathbf{X}(t)$ is the evolution of some initial orbit $\mathbf{X}(0)$ in the phase space, with time, then

$$\lambda_1 = \lim_{t \rightarrow \infty} \lim_{\varepsilon \rightarrow 0} \frac{1}{t} \ln \frac{|\delta \mathbf{X}_\varepsilon(t)|}{|\varepsilon|} \quad (2)$$

where

$$\delta \mathbf{X}_\varepsilon(t) = \mathbf{X}(t) - \mathbf{X}_\varepsilon(t) \quad \text{and}$$

$$\varepsilon = \mathbf{X}(0) - \mathbf{X}_\varepsilon(0)$$

for almost all initial difference vectors $\varepsilon = \mathbf{X}(0) - \mathbf{X}_\varepsilon(0)$. We estimate λ_1 using the algorithm of Sano and Sawada [4], where λ_1 describes the divergence and instability of the orbits in phase space.

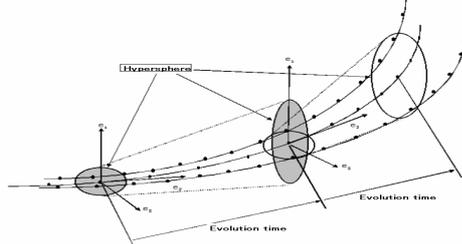


Fig.4 Method of calculating the Lyapunov exponent.

The initial 8,000 points of pulse wave data were taken as one window to calculate the largest Lyapunov exponent, λ_1 . In the next step, the window was shifted by 200 points and the exponent was calculated from the next window of 8,000 points. This procedure was repeated until the pulse wave data were exhausted. Three minutes of measurement yielded 36,000 data points. Therefore, we can obtain a $(36,000 - 8,000)/200 = 140$ -point time series of Lyapunov exponents, and with 1 min of measurement, we can achieve a $(12,000 - 8,000)/200 = 20$ -point time series of Lyapunov exponents. The size of the hypersphere used was 0.08. The variation of the largest Lyapunov exponent is a measure of the variation of the trajectory of the four-

dimensional attractor. The largest Lyapunov exponent is the divergence of the attractor trajectory and is an important value related to psychological indices [6], [7].

2.3. Outline of self-checking at the time of measurement

The subjects were asked to answer some simple questions to ensure normal measurement of pulse waves. This information was used to interpret the observed divergence in measured values. The questions were status-checking items, including physical condition and a simple assessment of the mood at the time of measurement. The answer was chosen from one of three available choices. Apart from these questions, the subjects were asked to describe their mood and condition at the time of the measurement in more definite terms. This enabled the person to identify factors that affect the divergence values. Because these records were made in a free-form style, key words alone could be used. However, when a subject is allowed to write freely, for example, about things that he or she had communicated to a friend, music he or she enjoyed, positive results in a sporting activity, and good or bad news that had been received, it is easier to study the relationship of those events with the divergence value.

The situations that elicit emotions such as delight, anger, sorrow, and pleasure, differ from person to person. For example, a condition that creates a suitable level of divergence, such as listening to music or having a conversation with someone, need to be recorded as data unique to that subject. In addition, extreme tension, fatigue, and low spirits also cause changes in divergence. Therefore, comparing the status recorded at the time of measurement with the corresponding divergence values helps a person to assess his or her own mental status. Table 1 shows the details of the questions with three levels of answers, and the freestyle description method. These responses and the simple comments on the subject's condition are stored so that they can be seen by clicking the corresponding divergence value on the graph.

Table 1. An example of entering the subject's state at the time of measurement: Status checking items

Status	Good	Normal	Poor
Sleep	✓		
Appetite		✓	
Health	✓		
Will to work		✓	
Mental health			✓
Current mood			✓

Freestyle reporting: The subject is also to enter a note on his or her condition at the time of measurement. These notes can be in the form of a descriptive comment on the subject's condition, keywords, and more.

Comment example 1: [Had a pleasant chat with a friend about hobbies.]

Comment example 2: [Feeling low after failing a test]

We plan to vary the simple questions described above according to the category (child, adult, or aged person) of the subject.

3. Relationship between the divergence value and cognitive psychology and the relevance of using fingertip pulse waves.

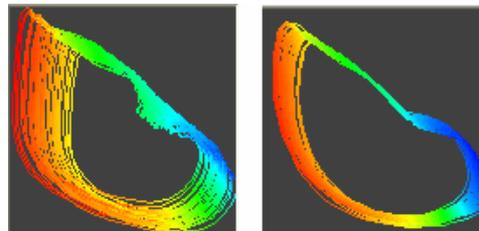
Biological systems are considered to be complex, fluctuating with chaotic characteristics. Although chaotic systems appear to be extremely complicated and to behave in a random and unstable manner, they in fact change according to deterministic rules. Biological signals emanating from humans, or vital signs, come in many types, such as the body temperature, blood pressure, and pulse rate. Fingertip pulse waves are biological signals that produce time series data with chaotic characteristics. Moreover, unlike cardiac waves, fingertip pulse waves contain various types of information, such as that from the nervous system. In the field of psychology, several methods have been suggested as indices for assessing mental health. However, these methods are generally subjective, and therefore lack objectivity. Questionnaires have often been used as relatively simple psychological tests, and the measurement and analysis of brain waves can objectively assess the neurological state at the time of measurement. However, the measurements are not simple, and the analysis methods are not yet capable of analysing detailed psychological changes. Another possible method for measuring biological information is to analyse the R-R intervals of heartbeats and pulse waves. However, no analysis has attempted to take into account the chaotic characteristics of biological information.

The Lyapunov exponent is a property of chaotic systems that expresses the attractor and represents the “divergence” of the attractor trajectory. We focussed on the Lyapunov exponent, which had not been quantitatively evaluated as an index of psychological changes in humans, and investigated its relationship with dementia and communication skill (an ADL index) in aged persons [1], its relationship with error rate at work [8], its relationship with diurnal changes and indices of cumulative fatigue in employees [5]–[7], and the time series fluctuations in divergence in 0- to 5-year-old children and the effect of parental affection towards the children [9].

It became clear that suitable functioning and harmony of the sympathetic nerves, which are related to the adaptability to the external environment and the society, and flexibility, spontaneity, and cooperativeness of the mind, are important for humans to live in a mentally healthy state. These values were related to the largest Lyapunov exponent obtained by nonlinear analysis [6], [7]. The largest Lyapunov exponent that represents the time series variation in the attractor trajectory is defined as “divergence.” When this value remains continuously low (i.e., when a long spell with low divergence persists), the person would have a low ability to adapt to the external world in their daily life and would be incapable of maintaining a mentally healthy state. However, an extremely tense or stressful state persists when the divergence remains continuously high. A mentally healthy state also cannot be maintained in this situation. Normally in humans, a healthy state is the condition in which constant variation occurs in the divergence. Emotions are a part of being human, and these are believed to cause the variation in divergence.

Physical immunity is crucial for the maintenance of human health, and lowered immunity causes various diseases. Therefore, to prevent the lowering of physical immunity and to increase resistance and prevent diseases, we pay attention to what we eat, rest, take medicines, and train to improve our stamina. However, mental toughness, as reflected in the ability to communicate in a positive manner, willingness to do the given jobs, and the ability of mental toughness to withstand drastic changes in the external world, are also very important. We can call these “mental immunity,” but no methods have been developed to scientifically investigate such immunity. We analysed fingertip pulse waves through nonlinear analysis and examined their relationship with various psychological indices, and found that the largest Lyapunov exponent obtained through chaos analysis, which corresponds to the “divergence” of the attractor, was closely related to mental immunity. It was also demonstrated that this value was closely linked with functioning of the sympathetic nerves of the autonomic nervous system.

For humans, a mentally healthy condition means having the ability to cope flexibly with external changes in “divergence.” This can be considered mental flexibility, or mental immunity, in contrast to physical immunity. Mental immunity represents adaptability to the external changes in that a person has to face in his or her everyday life, including one’s communication ability, ability of self-expression, and suitability of psychological flexibility. Whilst expressing themselves, humans skillfully fend off various kinds of changes, contacts, and assaults from the external environment, and deal with or cope with them in some cases. This is the essence of mental immunity. Change occurs constantly in day-to-day life. “Divergence,” which represents a change in the state of mental immunity, is a critical index. At the same time, divergence varies depending on various conditions of the person. For example, a long time period without “divergence” suggests that the person is not in a normal state. Figure 5 shows the attractors of a mentally healthy person and a patient with depressive psychosis. The depressed person has low divergence. Figure 6 shows the attractors of two dementia patients; both figures contain data from elderly subjects. The divergence becomes smaller as dementia advances. (CCI software was used.)



Healthy person

Depressed patient

Fig. 5. Attractors of a healthy person and a depressed patient prepared from 30-sec measurements.

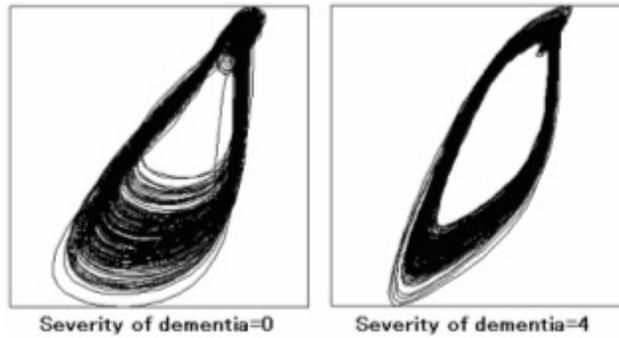


Fig. 6. Attractors in old people with dementia of severity 0 and 4.

In a normal state, the divergence level constantly fluctuates. During times of extreme tension and stress, continuously high divergence will exist. Afterward, however, a mentally healthy person naturally finds a way to relax, which brings the divergence back to its normal state. However, a low level of divergence would continue when a person is in a depressed state or when age-related dementia is present. This suggests that the person is incapable of bringing back the divergence to its natural level on his or her own, indicating a lowered adaptability to the external environment.

Table 2 shows the comparison of the midday Lyapunov exponent and the fatigue indices during the work day of employees at a particular company. The table indicates an inverse correlation of -0.7 or greater between the midday Lyapunov exponent and state of depression or anxiety of the employees.

Table 2. Coefficients of correlation between the Lyapunov exponent (at midday) and components of mental fatigue in company employees

	Willpower decrement	Anxiety	State of Depression	Accumulated Tiredness	Lyapunov exponent midday
Willpower decrement	1	0.7235	0.7539	0.7496	-0.6385
Anxiety	0.7235	1	0.8455	0.9358	-0.7279
State of depression	0.7539	0.8455	1	0.842	-0.7014
Accumulated tiredness	0.7496	0.9358	0.842	1	-0.6305
Lyapunov exponent midday	-0.6385	-0.7279	-0.7014	-0.6305	1

The experiments conducted thus far have shown that a suitable variation in the Lyapunov exponent is related to an adaptability to the external environment, which is necessary for a person to live a healthy life. This value fluctuates constantly. Therefore, by monitoring the fluctuations to detect states with very little or too much variation, a person could, on his or her own, assess one's mental health. Below, we outline this mental health self-check system.

4. Outline of the mental health self-check system

4.1. Development of the measuring equipment

A device that is easy to use and gives minimum burden on the subject is needed to measure the pulse waves. It is not possible to check the mental health of a person through just one round of measurements. For these reasons, the device must be convenient to use. We took note of the fact many people often do their work with PCs, and therefore developed a device that can make these measurements using a mouse. As shown in Figure 7, the pulse wave sensor is installed on one side of the pulse wave (PW) mouse; measurements can be made by simply touching the sensor with a finger. The mouse is connected to the PC through a USB port and can also be used as an ordinary mouse (patent pending).

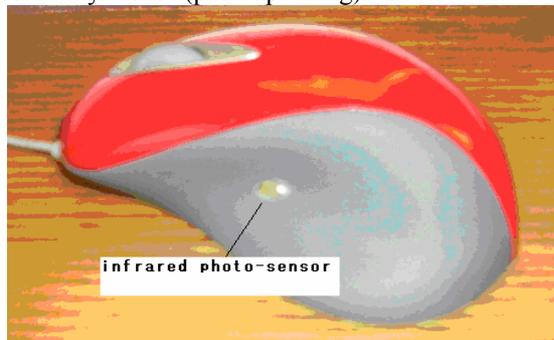


Fig. 7. Mouse for measuring pulse waves.

Software installed on the PC starts and ends the measurements, and sets their duration.

4.2. System configuration

Figure 8 is a configuration diagram of the mental health self-check system. A PW mouse icon is created on the PC desktop, and the menu screen is displayed by clicking on the icon.

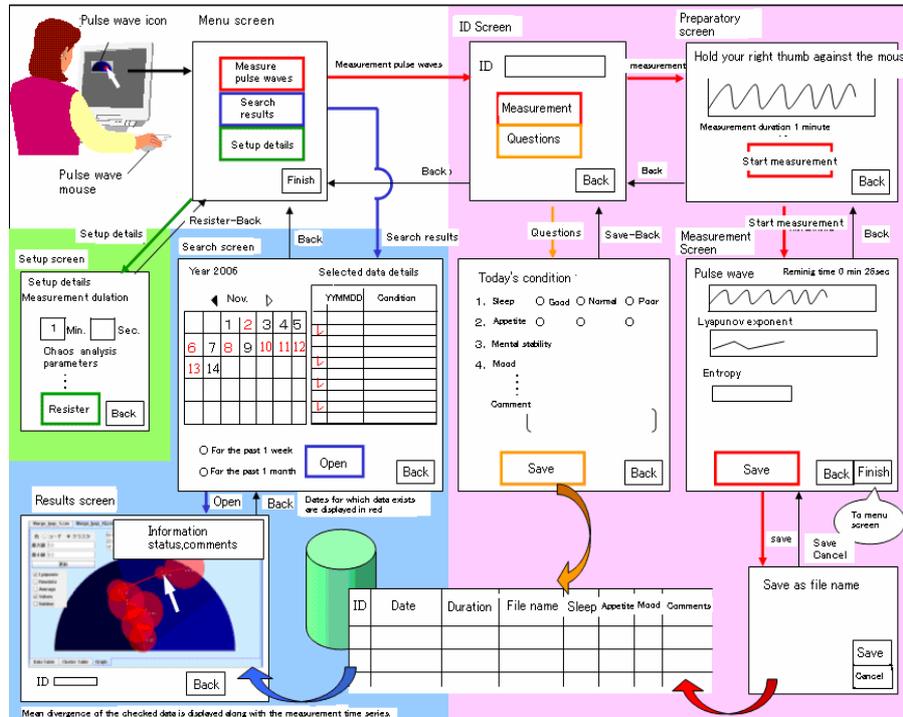


Fig. 8. Flow diagram of the mental health self-check system (fully PC based).

The screens are structured so that clicking the PW mouse icon on the screen displays the initial screen of the program. The user can then navigate to the measurement, search, or setup screens, where the respective functions can be performed. In the measurement section, the subject inputs his or her ID, does the data collection, and answer questions. In the pulse-wave-measuring step, the Lyapunov exponent is computed, and after it is confirmed that the pulse waves are being captured normally, the data are saved. After the pulse wave measurements are completed, the user answers the questions and the answers are saved. When the pulse wave measurements and the inputs of the answers to the questions are completed, the system moves back to the initial screen. If the person clicks the search button, a list of saved data is displayed. The constellation graph can be displayed by clicking the desired date. From one to a maximum of ten dates can be selected. The display is prepared according to the sequence of the selected measurement dates. The person can self-check his or her mental status by examining the variation in the Lyapunov exponent and the text displayed at the top right in the constellation graph. The menu screen display is structured so that various parameters can be changed based on the initial screen setup. When default values are used for the safe operation of the system, the displays are based on the fixed settings.

4.3. Display of results and constellation graphs

Figure 9 presents constellation graphs. The change in divergence is displayed according to the selected dates of measurement. When a large divergence occurs, the line moves to the right, and when it is small, the line shifts to the left.

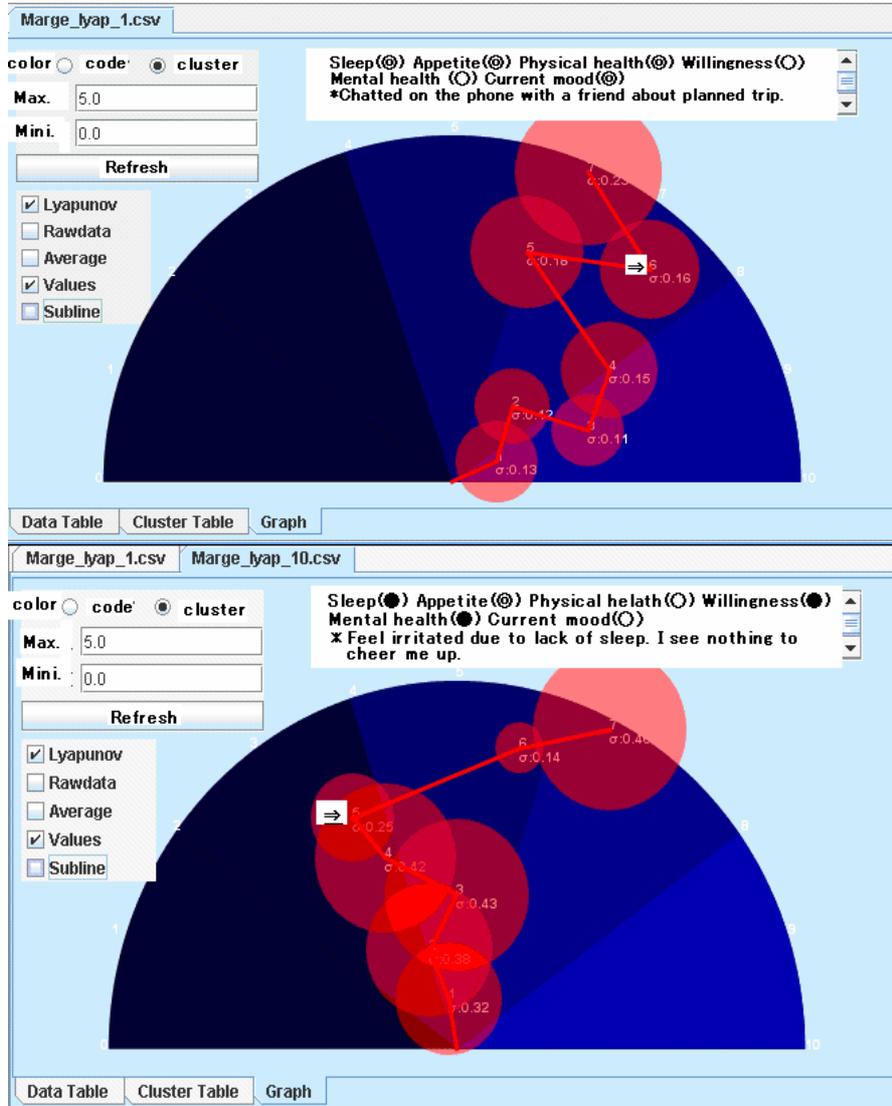


Fig. 9. Examples of time series constellation graphs.

The results of seven measurements are displayed simultaneously. The circles around each point represent the standard deviation. When the cursor is placed over

the centre of a circle, the display in the box at the top right changes to show the information that corresponds to that measurement.

As noted in Section 2.2, 1 min of pulse wave measurements yields a 20-point time series of Lyapunov exponents. The mean and standard deviation are calculated. Each mean value of the Lyapunov exponent is converted into an angle, with 180° assigned the maximum value of 5.0 and 0° given the minimum value of 0.0, and displayed as a vector on the constellation graph. For displaying the data of n number of measurement dates, the radius r of the full semicircle is divided into n equal sections, and n equidistant concentric semicircles are drawn. The sequence of the time series is set from the centre towards the outside. The n Lyapunov exponents of this time series are converted into angles and represented as a series of connected vectors. The point at which a vector intersects the corresponding semicircle is taken as the origin for the next vector to obtain the type of graphs shown in Figure 9

5. Future plans and the effectiveness and drawbacks of the system

We have described a system in which the entire procedure is performed on a PC. In the future, we plan to develop this into an Internet-based system, with software kept on a server and launched by clicking an icon. Pulse-wave data, answers to the questions, and the descriptive comments would be stored under the person's ID in a database created on the server. Thus, data from a larger number of users can be collected to facilitate a more effective analysis.

To encourage more people to use the system, we are also considering incorporating the PW mouse into cell phones to allow the display to be viewed on cell phone screens. When using cell phones, the data processing would be achieved by downloading the software and searching the database via the Internet in the I-mode. The use of Internet-based methods would produce a vast amount of data, and privacy-protection issues would have to be thoroughly addressed.

The following are conceptual diagrams of the fully PC-based system, an Internet-based system, and a cell phone-based system.

- A. Fully PC-based system (See Figure 8).
- B. Internet-based system.

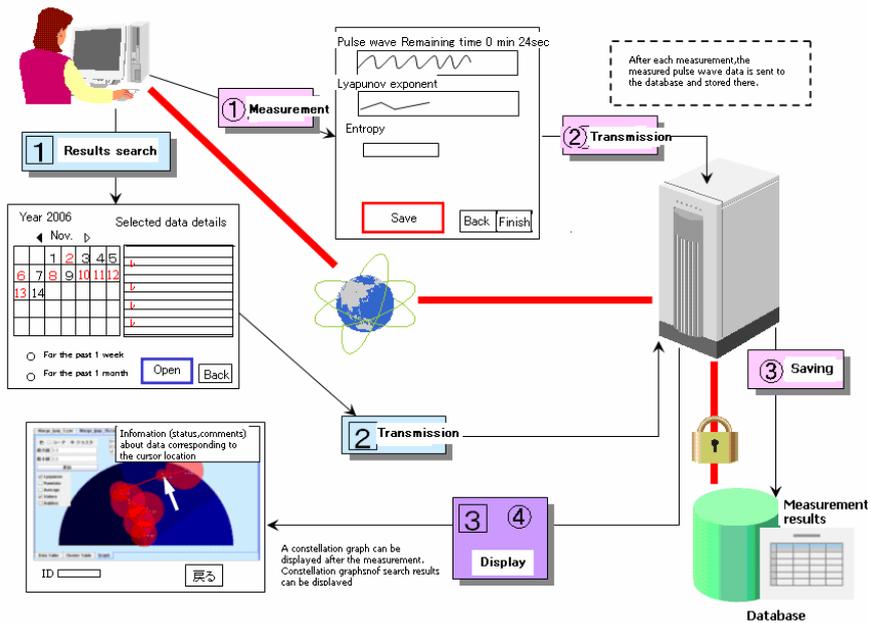


Fig. 10. The Internet-based system.

C. Cell phone-based system.

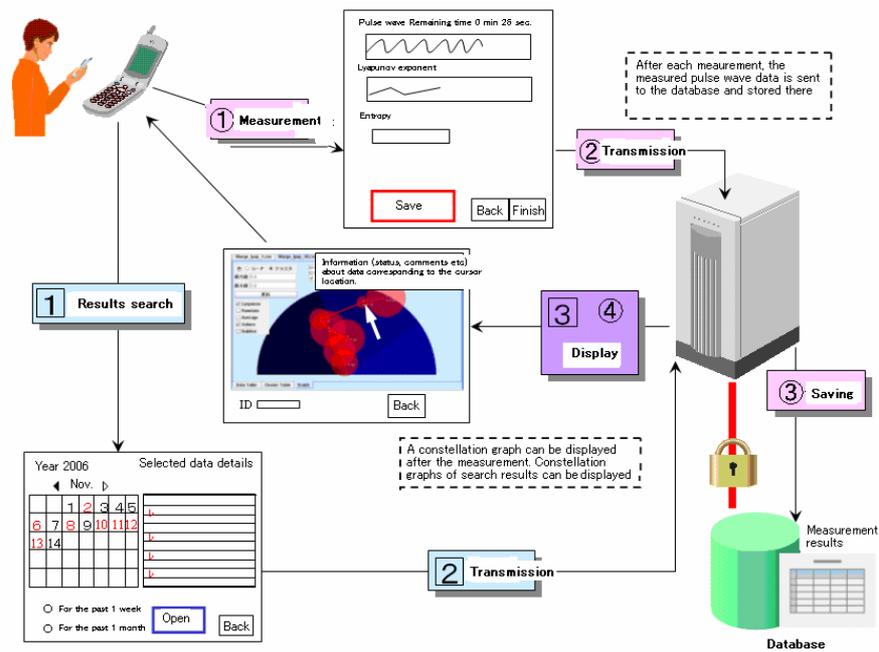


Fig. 11. The cell phone-based system.

6. Conclusion

The system described above allows a person to self-check his or her mental health level and also enables self-management. Appreciating the mental states that give rise to high or low divergence in day-to-day life would help to maintain mental health. We firmly believe that when a person can no longer exercise self-control, the early detection of depressive psychosis or dementia through consultation with a counselor or psychiatrist, and the prevention of deteriorating mental health, would be possible. Moreover, if data related to the psychological indices of persons could be sent and received through a network, this could lead to previously unknown communication innovations. Nevertheless, we must pay great attention to data management because of recent problems in protecting personal information. In terms of the system's effectiveness from a research point of view, we are certain that accumulating vast amounts of information would be useful in various future studies on pulse waves.

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